HARVARD | BUSINESS | SCHOOL

9-705-471
REV: MARCH 26, 2007

FELIX OBERHOLZER-GEE
DENNIS YAO

Game Theory and Business Strategy

In the experience of many managers, important business decisions are often very complex. In part, it is difficult to make strategic decisions because companies are increasingly interdependent and the outcomes of many managerial decisions are highly uncertain. Game theory is an analytic tool that allows decision makers to deal with *interdependence* and *uncertainty*. The theory was codified in the 1940s by John von Neumann, a mathematician, and Oskar Morgenstern, an economist, but it took decades until game theory could be applied to problems that were of interest to managers. Today, game theory is widely used in business and policy, and knowledge of game-theoretic principles belongs in the toolbox of any strategist. In this note, we describe the game-theoretic approach to analyzing complex business situations and use the theory to study a common business transaction—the decision to sell off a plant.

Analyzing a Strategic Interaction

The results of almost any strategic action depend on how other companies react to the action. The more the success of a strategy depends on the anticipations and reactions of competitors, suppliers, or customers, the more valuable it is to use game-theoretic analysis. This type of analysis involves three steps.

1. Problem formulation

   At the outset, it is important to be clear about the question we want answered. For example, "Is it a good idea to lower prices for our products?" is not a good question because much depends on the definition of "a good idea." "Do sales increase if we lower our prices?" or "Do profits increase if we lower our prices?" are better questions because they make it clear what we want to know. Having a precise question also helps because we can disregard factors that do not influence sales or profits.

2. Model building

   Once we have a precise question, we are ready to build a game-theory model. We do this by answering five questions, as follows:

   - Who are the *players*?
   - What are the *actions* available to the players and what is the *timing* associated with those actions?

Professors Felix Oberholzer-Gee and Dennis Yao prepared this note as the basis for class discussion. The example used in this note is adapted from a BA901 project by J. Alvarez and M. van Berge Henegouwen.

- What *information* is available to each player?
- What are the strategies?
- What are the payoffs?

Game theory is about strategic interaction among *players*, so it makes sense to determine at the beginning who the players are. Players might be competitors, suppliers, or customers. Next, we list the *actions* that these players can take. For example, our competitor can introduce a new product and can sell this product at a low or at a high price. It is useful to determine the order in which players take actions in a game so that we can see what information each player has when the player makes a decision. For example, in an industry with short development cycles, we might know if our competitor has introduced a new product before we decide whether or not to launch ours. In contrast, with long development cycles, we might have to decide on our product launch before we learn what our competitor has done. These types of games are referred to as *sequential-move* and *simultaneous-move* games, respectively.

In a next step, we list all possible *strategies* that a player can choose. A strategy is a plan that describes which actions a player will take given the actions of other players. In the simultaneous-move game, introducing a new product *and* selling it cheaply is one strategy. Not introducing a new product is another.[1] With two actions (product introduction and price), our competitor can play three different strategies in the simultaneous-move game.[2] It is important to remember that a strategy is a *complete plan of action*; it tells you what to do in every contingency of the game. Once we list the strategies for all players in a game, we calculate *payoffs* for each combination of strategies. For example, what are the profits for us and our competitor if he introduces a new product and sells it at a high price while we have no new product? Or what are the profits if the competitor introduces a new product at a high price and we offer our new product at a low price? Comparing the payoffs for combinations of strategies tells us what our competitor is likely to do and how we can best anticipate and respond to his actions. That is, we have built a model of strategic interaction.

3. Decision

Game-theoretic analyses are valuable because they provide intuition for the likely outcome of strategic interactions. With the modeling exercise completed, it is a good idea to test the intuition against the reality of the business situation in which we are interested. Does the recommended strategy make sense? Did we list all the relevant actions? Can we be sure that the payoffs were specified correctly?

To see what these three steps look like in the context of a business decision, we next describe a planned asset sale.

---

[1] In this case, the price decision is moot.

[2] The strategies in a game critically depend on the information we have when we make decisions. To see this, consider a game where two car manufacturers simultaneously decide to produce either a small car or a big car. Once they learn the decision of the other firm, they set a high price or a low price for their car. The first stage of the game has four outcomes: (small, small), (small, big), (big, small), (big, big). A strategy tells a manager what to do in the first stage and how to proceed at the second stage given any one of the four possible first-stage outcomes. For example, a strategy is (small; if small then low, otherwise high), which means build a small car at the first stage, then always charge a high price in the second stage except when both firms decided to build a small car.

Application: The Sale of United Cement's B Plant

United Cement Company had been approached by a firm that wanted to acquire United's B plant, the smaller of two cement plants. The B plant was currently idle. The prospective buyer had no interest in the cement business and planned to tear down the plant and use the land for other purposes. It was generally believed that the buyer is unlikely to pay very much.

Donna Duncan, head of United Cement Company's Strategic Planning Group, asked John Morgenstern, a recent graduate from business school, to prepare a preliminary assessment of the sale for the next morning's planning group meeting. John was pleased to be asked to take on this assignment but was concerned that the time frame would put a significant limitation on the amount of information he could collect to support the analysis.

Morgenstern knew the following facts:

- United had an 8,000-ton per year plant and a smaller 2,000-ton plant. The smaller plant was currently idle. United's only competitor was Associated, which operated a single 20,000-ton plant.
- At current prices there was significant overcapacity. Over the last five years each firm had been producing at 50% of total capacity (Associated at 10,000 tons and United at 5,000 tons). There was no significant difference in the quality of the cement from any of the plants, and all three plants were equally efficient.[3] Relative to variable costs, fixed costs were negligible. The current market price for cement was $5 million (U.S.) for each thousand tons of cement sold. Variable costs were $2 million per thousand tons.
- Marketing had indicated that lowering the price to $4 million for each thousand tons while the other firm maintained its price at $5 million would likely shift half of the other firm's demand to the firm with the lower price. If both firms lowered the price to $4 million, each would maintain its original share of overall demand. United's marketing team believed that the marketers at Associated held the same view. Unfortunately, United's marketing was unable to commit to an estimate of how much overall demand would change with a lower price.

Based on this information about demand responsiveness, cost, and capacity, Morgenstern made a number of profit calculations, which are summarized in **Tables A** and **B**. For the purposes of his initial calculation he assumed no overall demand increase with lower price.

Table A Profit Implications of Various Pricing Actions in Current Situation

	United ($ million)	**Associated ($ million)**
Current ($5 million) pricing	15.0	30
United lowers to $4 million and Associated maintains price	20.0	15
Associated lowers to $4 million and United maintains price	7.5	25
Both lower price	10.0	20

Source: Casewriters.

[3] In reality, each plant needed to produce at some modest rate to be efficient. At the various levels of production that were relevant here, variable costs were the same.

Table B Profit Implications of Various Pricing Actions after Sale

	United ($ million)	Associated ($ million)
Current ($5 million) pricing	15.0	30
United lowers to $4 million and Associated maintains price	16.0	21
Associated lowers to $4 million and United maintains price	7.5	25
Both lower price	10.0	20

Source: Casewriters.

United's B plant had not been run for a few years and, while there were disagreements within the firm about what to do with the plant, few now believed that overall demand for cement in this region was likely to increase. At first, Morgenstern thought that it was only necessary to determine an appropriate market value for the plant and the land. But when Morgenstern asked some of the more experienced members of the planning group why the plant had not been previously sold, he was told that the capacity might have played a role in pricing dynamics. Despite chronic overcapacity in the industry, pricing had been remarkably stable. Each company currently had market shares in direct proportion to its capacity. Morgenstern wondered if the sale of this idle plant would impact the price of cement or United's share of the market.

To get at the sale question, Morgenstern was convinced that he had to understand more about pricing dynamics in the industry. Perhaps game theory could be of use.

A Bimatrix Game Analysis of the Sale Problem

The first step in developing a game-theoretic analysis is to ask a precise question. Morgenstern posed the following nested questions: What is the correct pricing strategy for United with and without the sale of the plant and, given the answer to that question, at what price should the plant be sold?

To analyze these questions, it is useful to build a simple game-theoretic model. Even with limited information and a number of simplifying assumptions, game theory can produce much strategic insight. We start by answering the questions listed at the beginning of this note. Cement is a local market, mainly because it is very expensive to ship large quantities of cement. Thus, we only need to consider the two local players, United and Associated, in our model. What are the actions that United and Associated can take? What interested Morgenstern was how the two companies set prices and whether or not United should sell the idle plant. These are the actions that we will consider in this model.

To develop our intuition about pricing decisions in this market, we consider an extremely simple representation. Suppose the players' options are to maintain their current price of $5 million or choose a lower price of $4 million. We assume that both companies choose their prices simultaneously and are committed to that choice. What are the profit implications (the payoffs) of this *simultaneous-move single-shot* game? From **Table B** we can construct the following:

Table C

(United's Profits)	Associated Maintains Price	Associated Lowers Price
United maintains price	15	7.5
United lowers price	16	10.0

Source: Casewriters.

No matter what Associated does, United makes a higher profit by choosing the lower price. Game theorists say that United's "lowers price" strategy *dominates* United's "maintains price" strategy. A strategy is strictly dominated if the payoff from playing that strategy is strictly less than the payoff from other strategies for all possible strategies that the other player can choose. One important way to solve games is to *eliminate dominated strategies*. In our case, United maintaining its price is an example of a dominated strategy.

The one-shot analysis suggests that United should lower price. But this does not answer the question of whether the plant should be sold (and at what price), as United cannot know whether Associated will maintain or lower its price without analyzing Associated's incentives (payoffs)—especially since one might expect Associated to also lower its price.

We can add Associated's payoffs to the same table to get a *bimatrix game* representation of the simultaneous-pricing competition between United and Associated. Here United's payoff to a given set of actions (e.g., Associated maintains and United maintains) is listed to the left, and Associated's corresponding payoffs are listed on the right. Payoffs are typically written on the same line, but here we have staggered them to make the table easier to read.

Table D

(United's Profit)	Associated Maintains Price	Associated Lowers Price
United maintains price	15, 30	7.5, 25
United lowers price	16, 21	10.0, 20

Source: Casewriters.

When United analyzes Associated's pricing decision, it becomes clear that Associated's action to "maintain" dominates its action to "lower." United should also think that Associated will figure this out. Further, United ought to suppose that Associated will engage in a similar analysis of United's decision and come to the conclusion that United has a dominant strategy to lower price. Both expect each other to play the dominant strategy, with the result that United lowers price while Associated maintains its price. In this case, United earns $16 million, while Associated earns $21 million. If we compare United's new profit after the sale of the plant to its current (presale) profit of $15 million, we see that United would be willing to pay a buyer to take plant B off its hands!

This was a pretty startling twist to the analysis, but the result came from a very simplified model of the pricing interaction. Before going to boss Duncan, Morgenstern needed to determine the nature of the strategic insight and assess its relevance to the real situation (step 3 in our list). What had he learned about the competitive interaction that he could take to the actual setting?

A key question is why Associated is willing to maintain its price even if it expects United to lower its price. First, let's look at the generation of the (16, 21) pair of payoffs. Why does United's profit only increase by $1 million if United lowers and Associated maintains its price? Marketing indicated that half of Associated's demand would come to United if United lowered its price while Associated maintained. Should not more profit be generated from this increase in United's sales? The answer is that although half of Associated's previous buyers (5,000 tons of demand) would like to purchase from United (making a total of 10,000 tons of demand), United cannot supply all of the demand because it only has 8,000 tons of capacity. This reduction of capacity is due to the *sale of the idle plant*. Because Associated now only loses 3,000 tons, it is willing to maintain its price, whereas if it lost 5,000 tons, it would prefer to lower its price.

The sale of the idle plant is effectively a *credible commitment* by United not to steal too much demand if it has a lower price than Associated. This commitment changes the nature of the strategic interaction by reducing Associated's incentives to lower price. It turns out that this type of commitment is a common feature of many strategic problems, especially those involving small entrants that challenge dominant incumbents. It goes under the moniker "judo strategy." Game theory allowed Morgenstern to discover this strategy.[4]

We make three additional comments on this analysis. First, game theory (generally) requires each player to solve the other player's problem. One's opponents are given credit for being as smart as you are. Second, it is essential to check at the end of the analysis if the intuition that the model provides can transcend the simplifying assumptions that were made for model-building purposes. For example, the one-shot game does not allow for a pricing adjustment after a competitor's price is observed. Suppose that Associated could change its price after it observed United's lower price. Would it do so? The answer in this case is no. Therefore, that assumption does not undermine the value of the insight from the model.

Third, we did not examine the pricing dynamics in the original game, relying instead on the $5 million price (by both) as the given outcome from the current situation. A one-shot game analysis of the pricing dynamics of the current situation before the sale of the plant suggests that United has a dominant strategy to lower its price. Associated anticipates this dominant strategy and chooses its best response to United's lower price, which in this case is to lower price as well. Thus, the prediction of the one-shot game with the idle plant B is that both lower price. This is obviously not what we observe in reality, where both companies maintain price. How did the managers achieve this outcome? One possibility is that the multiple rounds of play explain the difference. United and Associated compete with each other week after week. This competitive situation is a good example of a *repeated game*. If United decided to maintain price in the first round, with the credible threat to lower the price next round if Associated decided to lower its price, what would Associated's best response be? If Associated believes that United will maintain its price, it has an incentive not to lower. By using the threat of lowering, United can facilitate cooperation and gain $15 million (every year) rather than the $10 million it would get in the one-shot game.

---

[4] The strategic role of commitment is discussed in Pankaj Ghemawat, *Commitment: The Dynamic of Strategy* (New York: The Free Press, 1991). David B. Yoffie and Mary Kwak analyze Judo Strategy in "Judo Strategy: Turning Your Competitors' Strength to Your Advantage" (Boston: Harvard Business School Publishing, 2001).

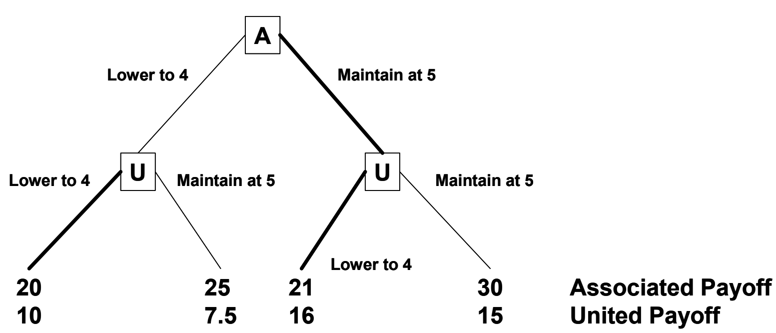
A Game-Tree Analysis of the Sale Problem

Because game theory was not well understood by the management at United, Morgenstern thought about other ways in which he could make the point that selling plant B and lowering price was the right strategy for United. He decided to build a *game-tree* analysis to buttress the bimatrix analysis, which showed that Associated would not respond to the sale of plant B with a price war.

A game tree consists of a set of decision nodes that are connected by branches (see the game tree below). Branches that extend from a decision node to other nodes represent the entire set of possible actions available for that decision.[5] Nodes and branches are ordered with the first decision (the *root*) placed at the top of the tree and later decisions placed further down. The tree terminates in a set of end points, each of which is associated with the payoffs each player would receive if that sequence of actions was played. Because the structure of the tree is sequential, later branches do not connect to earlier nodes, so there are no closed loops. Note that the actions of all players are represented in the game tree so that the solution necessarily involves considering each player's best choices, not just your own. Because game trees depict a sequence of decisions, such trees are the most natural tool to depict a sequential-choice game.

The game tree below models the pricing game that would follow the sale of plant B. In this model Associated is assumed to make the first move—either lowering the price to $4 million or maintaining it at $5 million. This decision is represented by the top node in the tree and the two branches extending from the top node. The next decision is United's response, with possible actions of lowering the price to $4 million or maintaining it at $5 million. Because United observes Associated's action before making its own pricing decision, its actions are contingent on Associated's move. Hence there is a separate United decision node after each of Associated's possible actions.[6] The payoffs given at the bottom of the tree are specific to each sequence of actions. For example, if Associated chooses to maintain its price at $5 million and United responds by lowering its price to $4 million, Associated and United will earn $21 million and $16 million, respectively.

Figure A



Source: Casewriters.

[5] Uncertainty can be represented by a "chance" node with associated branches that represent the possible resolutions to the uncertainty. Because chance is not "strategic," each of the branches emanating from a chance node is assigned a probability representing the players' assessments of the likelihood of each resolution. For example, one could have a chance node and branches representing the observable demand response to particular prices (e.g., high demand with probability 0.4 and low demand with probability 0.6).

[6] We have suppressed a more complex discussion of how to model a game tree in which actions are taken simultaneously (e.g., the previous action was not observed prior to the choice). In such a case, it is helpful to keep track of what is observed using the concept of an "information set." See, for example, A. Dixit and S. Skeath, *Games of Strategy*, 1999, for details on how this would work.

Game trees can be solved through *backward induction*. Backward induction entails figuring out the best actions for each of the decision nodes that are lowest in the tree. For the left United decision node (i.e., the node connected to Associated's action of lowering price to $4 million), the best action is to lower price to $4 million because from this node United would get a $10 million payoff from lowering but only a $7.5 million payoff from maintaining price at $5 million. For the right decision node, the best action for United is also to lower price ($16 million versus $15 million). Now consider the next higher decision node, which is Associated's pricing decision. In making this decision Associated anticipates United's optimal action. That is, Associated expects United to lower price if Associated lowers price and to also lower price if Associated maintains price. Given this analysis of United's optimal actions, Associated compares its two alternative actions: if Associated lowers price it expects to earn $20 million, and if Associated maintains price it expects to earn $21 million. Associated, therefore, chooses to maintain its price at $5 million. This game-tree analysis provided Morgenstern a slightly different way to present the logic in support of selling plant B.

Conclusion

Interdependence of strategies is a common feature of competitive dynamics problems. Yet while it is easy to acknowledge the need to consider the strategies of competitors, it is not easy to analyze such interdependencies in a disciplined fashion. Game theory provides a set of techniques that allows the strategist to do so. It forces a strategist to analyze a competitor's problem as if it were her own and to recognize that the competitor is thinking strategically as well. It provides a framework for developing, testing, and even explaining intuition about strategic interactions.

A strength of game-theoretic reasoning is that it can sometimes lead to insights that seem counterintuitive. The sale of United's B plant is a good example. The sale benefits the company because, with less capacity, United cannot steal as much demand from Associated as it could if it had greater capacity. United can then reduce its price in expectation that its competitor will maintain his price. In many business situations, flexibility is valuable. Consequently, one might have expected that United is better off if it has the option to start producing cement at plant B. There are many managerial books that recommend maintaining flexible production capacity to maintain "real options," especially if the direct cost of maintaining flexibility is very small. However, in United's situation, the intuition that companies are better off if they have more options is wrong. Analyzing strategic interdependence through game theory allows us to see that the strategic sale of the B plant improves United's profits precisely because it reduces the options of the cement producer.